Graphical & Statistical Analysis for Monitoring Estimates of Change at the Management-Unit Level

Version 2.0

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Executive Summary

This document describes the graphical and statistical analysis procedures for monitoring estimates of change at the management unit level. Monitoring involves comparing observed change to predicted change. Observed change is based on a sample of permanent and inconspicuous plots that are established and re-measured in a management unit. We assumed the plots were a simple random sample of the management unit and independent of the Vegetation Resources Inventory (VRI) plots. The option of monitoring plots that use the VRI sample locations is not considered in this report.

This document also demonstrates with simulated data how to estimate change and its associated variance. It also shows how to make graphical and statistical comparisons of observed change with predicted change.

Table of Contents

1. INTRODUCTION 1
1.1 TERMS OF REFERENCE 1
1.2 BACKGROUND 1
1.3 DOCUMENT OBJECTIVES 1
2. GROUND SAMPLING METHODS 2
2.1 OVERVIEW 2
2.2 OBJECTIVE 2
2.3 TARGET POPULATION 2
2.4 MONITORING METHODS – TWO OPTIONS 2
2.5 SAMPLE SIZE 3
2.6 SAMPLE SELECTION 3
2.7 PLOT ESTABLISHMENT AND RE-MEASUREMENT 3
2.8 FIELD PROCEDURES 4
3. GRAPHICAL & STATISTICAL ANALYSIS 4
3.1 STATISTICAL ESTIMATION OF CHANGE 4
3.2 COMPARISON OF OBSERVED WITH PROJECTED CHANGE 4
4. APPENDIX I – CHANGE ESTIMATION 6
4.1 THEORY 6
4.2 DISCUSSION 7
4.3 EXAMPLE OF VARIANCE ESTIMATES FOR CHANGE 8
5. APPENDIX II – EXAMPLES OF GRAPHICAL & STATISTICAL COMPARISONS 10
5.1 OVERVIEW 10
5.2 SIMULATED DATA 10
5.3 GRAPHICAL ANALYSES 11
5.4 STATISTICAL COMPARISONS 12
6. APPENDIX III – DEFINITIONS 14

List of Tables

Table 1. Example data and calculations of change estimates 9

List of Figures

Figure 1. Measured volume versus age. 11
Figure 2. Actual versus predicted yield. 11
Figure 3. Difference between actual and predicted 5 years growth from establishment to first re-measurement. 12
Figure 4. Confidence intervals (95%) on average differences by age class. 13

1. INTRODUCTION

1.1 TERMS OF REFERENCE

This report was prepared under contract for the Ministry of Forests (MOF) Resources Inventory Branch (RIB) by Eleanor McWilliams, *MSc, RPF*, A.Y. Omule, *PhD, RPF*, and Bill Warren, *PhD*, of J.S. Thrower & Associates Ltd (JST). Other JST team members were Ian Cameron, *MSc, RPF*, Guillaume Thérien, *PhD*, and Jim Thrower, *PhD, RPF*. This paper has been reviewed by the Expert Review Panel (ERP) that includes John Barker, *PhD, RPF*, Jim Flewelling, *PhD*, Kim Iles, *PhD*, Peter Marshall, *PhD, RPF*, and Don Munro, *PhD, RPF*.

1.2 BACKGROUND

In 1998 the MOF awarded a 2-year contract to JST to develop growth & yield monitoring protocols. The second year (2000) of this contract (Schedule A) involves three tasks to be completed:

1) *Conceptual sampling methods* – Develop and recommend conceptual sampling methods to estimate change[1] at the provincial level and monitor estimates of change at the management unit level. This task was completed in December 1999 with submission of the final report. [2]

2) *Detailed procedures* – Develop estimators, procedures, standards, and field protocols based on approved conceptual methods. Demonstrate the use and application of the recommended methods.

3) *Pilot strategic sampling plan* – Complete a strategic sample plan for a pilot study of change inventory design at the provincial level, including objectives, desired products, and a general strategic direction for implementing the pilot project.

This report, in part, addresses Task 2. The remaining deliverables for Tasks 2 and 3 will be reported separately.

1.3 DOCUMENT OBJECTIVES

This document describes the graphical and statistical analysis procedures for monitoring estimates of change at the management unit level. These methods focus on monitoring change estimates in timber attributes used in timber supply analysis.

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[1] Definition of change and other terms are provided in Appendix III.

[2] J.S. Thrower and Associates Ltd. 1999. Conceptual sampling methods for change inventory and monitoring of vegetation resources version 2.0. Unpublished report. December 13, 1999.

2. GROUND SAMPLING METHODS

2.1 OVERVIEW

This section summarizes the ground sampling methods for the provincial change inventory as outlined in the earlier report titled, *Conceptual sampling methods for change inventory and monitoring of vegetation resources version 2.0.*[2] This section will provide background reference for the graphical and statistical analysis procedures to be described.

2.2 OBJECTIVE

The objective of the ground sampling is to:

> *Monitor (check) the projected growth or change in timber volume and other tree attributes (e.g., species composition and top height) at a management- unit level.*

The data used for monitoring should be independent of the data used to generate the projected growth. The projected growth is obtained from the inventory and growth models. The purpose of the checking is to determine the accuracy of change projections used in timber supply analysis and to identify large differences should they occur.[3]

2.3 TARGET POPULATION

The target population to be monitored will depend on the specific objectives of the management unit. This population should be adaptable to changes in the land cover classification over time. For example, change in stand volume may be monitored in a fixed geographic area such as a TFL or TSA through time.

2.4 MONITORING METHODS – TWO OPTIONS

Two options are proposed for monitoring estimates of change in management units:[2]

1. *Re-measure a subset of existing or planned VRI plots.*
   This option consists of two components:
   - A. *A subset of VRI plots must be re-measured to check the ability of growth models to track observed change in timber attributes from initial conditions.*
   - B. *Use an independent sample to do periodic checking of the initial conditions from which the change estimates were based.*
2. *Re-measure a set of plots that are independent of the VRI.*

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[3] The principle sources of error in change projections are: (1) error in the initial conditions (e.g., inventory attributes) used as inputs to models, and (2) error in the ability of the models to estimate future growth.

This option involves establishing and re-measuring plots that are independent of the VRI and models used to project change.

Both options are reasonable and the choice should depend on specific management unit objectives. For example, the first option may be preferred if a VRI already exists and the focus is to check the predicted change (i.e., growth) in timber volume over time. However, this option may not address issues requiring an independent sample of the landbase or the need to monitor non-timber attributes.

This report focuses on the second option (as instructed by the MOF). The MOF should consider developing statistical formulae and examples for the first option so that on those occasions when a completely independent sample is unnecessary, the VRI approach can be used.

2.5 Sample Size

A minimum sample size (30) has arbitrarily been suggested for plots in a management unit.[2] However, sample size should be determined for each management unit and will vary with the attributes of interest, natural variability, desired level of precision in estimates of change, and the level of confidence in existing predictions. Larger samples will be required in most cases to permit post-stratification of the data. Additional samples may be added in the future as needed.

2.6 Sample Selection

Monitoring samples can be selected with equal probability based on simple random or systematic sampling. It is preferable not to pre-stratify the target population but post-stratification may be done to examine specific portions of the population. Systematic sampling can be done from an ordered list (with a random start) or by using a systematic grid over the management unit.

2.7 Plot Establishment and Re-measurement

Plots should be established and measured in the initial target population over as short a time frame as possible (preferably within 3 years). If the target population increases over time (e.g., due to harvesting) then new plots are established proportionately in the new portion of the target population. All plots should be re-measured on a consistent cycle (e.g., every 5 years) to provide a common time increment for change observations. If the plots are re-measured at different cycles, then the observations would have to be adjusted to a common time increment based on some assumptions. Differences in the calendar age in which plots are installed can be accounted for by comparing volume (or other) estimates by stand age.

2.8 Field Procedures

Data definitions, standards, measurements, and field cards for the initial measurement and re-measurements are to be based on the VRI ground sampling procedures manual (with some additions). The major additions are a fixed-area plot for large trees (≥ 4.0 cm dbh), and mapping of polygon boundaries in the plots. Plots will be permanent and inconspicuous, and subject to operational activities. Details of the additions to these field procedures are described in the report titled *Vegetation Resources Inventory Change Measurement: Preliminary Field Procedures* (not yet completed). These field procedures may be modified depending on specific management unit objectives.

3. GRAPHICAL & STATISTICAL ANALYSIS

3.1 Statistical Estimation of Change

For each re-measured plot, change is estimated as simply the difference between the variable of interest at time 2 and time 1. Estimate of change for the target population (or portion of plots established in a given year) is simply the average plot change. The formulae for the estimates of change and their variance estimates under different sample selection scenarios are provided in Appendix I.

3.2 Comparison of Observed with Projected Change

3.2.1 Graphical Analysis

There are several simple and proven ways data can be plotted to provide a graphical comparison of actual and predicted values (Appendix II).

1. *Plot measured values versus age (or height) against a yield or site curve.*
   For example, plot volume/ha versus age (or height) against an aggregate volume curve for the entire target population. In this case, measured values will likely show considerable variation around the aggregate curve.
2. *Plot measured versus predicted values.*
   If predicted values are accurate, then most points should fall along the 1:1 line of correspondence for the unit the plot represents. Outliers can be flagged and examined to determine if any potential problems in prediction are indicated.
3. *Plot the difference (actual – predicted) versus age (or height).*
   This may indicate trends of over- or under-prediction for different age ranges. The same

absolute differences at young and old ages have different implications. Expressing differences as a percent of the predicted value can provide a better view of the differences over time and age (or height).

3.2.2 Statistical Analysis

Known simple statistical tests can be used in conjunction with graphical analyses. These are similar to the inventory audit procedures, whereby the difference between actual and predicted values is determined for each plot. An average difference and its associated confidence interval is then calculated. If this confidence interval includes zero, there is no significant difference between the actual and predicted values. In many cases, it may be necessary to post-stratify the data (e.g., by site) and calculate average differences and confidence intervals for each stratum.

3.2.3 Discussion

The graphical and statistical analysis methods presented here are intended as tools to examine the data for possible overall trends of over- or under-prediction; they are not meant to be definitive tests.

This first pass of the analysis should compare measured change with that predicted from the inventory and the growth model or other prediction system. If this analysis suggest over- or under-prediction, then possible sources of the differences should be identified. For example, when considering volume estimates, potential factors to consider as sources of mean error are the differences between the inventory inputs to the model and the actual stand attributes. Potential inventory attributes to examine include stocking, site index, treatment, species composition, stand structure, and pest and disease incidence.

4. APPENDIX I – CHANGE ESTIMATION

4.1 THEORY

The obvious estimator of change for a management unit where all plots are re-measured is simply:

$$\bar{y}_2 - \bar{y}_1$$

where:

$$\bar{y}_k = \frac{\sum_{i=1}^{n} y_{ik}}{n}$$

Both random sampling and systematic sampling (from an ordered list or grid) with a random start would provide an unbiased estimator of change, in the sense that the average over all possible realizations of the randomization process would equal the actual change. Another option to consider is purposive sampling with the intent of capturing the full range of stand conditions being projected on the management unit.

Following Warren (1994)[4] we can assume:

$$\mu_{ik} = \mu_{ik} + \phi_i + \psi_k + \xi_{ik}$$
$$y_{ik} = \mu_{ik} + \varepsilon_{ik}$$

where ϕ_i are plot effects, ψ_k are year effects, ξ_{ik} are analogous to the interaction in two-way analysis of variance, and ε_{ik} is the measurement error. With no loss of generality, it may be assumed that:

$$\sum_{i=1}^{N} \phi_i = 0 \qquad \sum_{k=1}^{2} \psi_k = 0 \qquad \sum_{i=1}^{N} \xi_{ik} = 0 \qquad \sum_{k=1}^{2} \xi_{ik} = 0$$

where N is the number of possible plots, and k is the number of years (here taken as 2). It will here be also assumed that there is no measurement error, i.e. $\varepsilon_{ik} = 0$, for all i,k.

If the same random set of plots is measured on both occasions, it can be shown that:

[4] Warren, W.G. 1994. The potential of sampling with partial replacement for fisheries surveys. ICES Journal of Marine Science, 54: 315-324.

$$E(\bar{y}_2 - \bar{y}_1) = \psi_2 - \psi_1$$

$$Var(\bar{y}_2 - \bar{y}_1) = \frac{4}{n}(\frac{N-n}{N-i})\sigma_\varepsilon^2 = \frac{4\sigma_\varepsilon^2}{n}$$

If the same set of purposively chosen plots is measured on both occasions, it can be shown that:

$$E(\bar{y}_2 - \bar{y}_1) = \frac{\sum_{i=1}^{n}(\xi_{s2} - \xi_{s1})}{n}$$

$Var(\bar{y}_2 - \bar{y}_1) = 0$, since one, and only one, sample exists.

Comparison between approaches should then be based on mean-squared error (MSE), i.e., variance plus the square of the bias. The MSE are:

Common random	Purposive
$4\frac{\sigma_\varepsilon^2}{n}$	$4\frac{(\sum_{i=1}^{n}\xi_{sk}^2)^2}{n^2}$

Estimates based on purposively selected samples would be more accurate (in terms of MSE) than common random samples if:

$$\frac{(\sum_{i=1}^{n}\xi_{sk})^2}{n} < \sigma_\varepsilon^2$$

This will depend on plot selection. A judicious choice of plots could result in estimates of small, if not negligible, bias and thus highly accurate estimates of change.

4.2 Discussion

At the management-unit level, unless there are some means to determine an appropriate purposive sample, the best strategy would be randomly or systematically selected re-measured plots.

One instance where a purposively chosen set of plots may be the preferred choice is the option of re-measuring a portion of the VRI plots and establishing independent audits of yield. The former could be checked from a set of permanent plots superimposed on a subset of the VRI locations. A sub-sample of the VRI plots could be purposively chosen to ensure that the range of initial conditions (model inputs) was adequately covered. This in effect, would be model

validation for the initial conditions of the management unit (or more strictly, the initial conditions represented by the VRI locations). Additional independent yield audits would confirm the accuracy of the model inputs.

4.3 Example of Variance Estimates for Change

For illustrative purposes, assume we have a set of *i = 1 to 10* re-measured plots. The y_{1i} are the measurements taken at time 1, and the y_{2i} are the measurements taken at time 2.

The estimate of average change is simply:

$$\bar{y}_2 - \bar{y}_1 = \frac{\sum_{i=1}^{10}(y_{2i} - y_{1i})}{10}$$

The estimated variance of $(\bar{y}_2 - \bar{y}_1) = \hat{Var}(\bar{y}_2) + \hat{Var}(\bar{y}_1) - 2\hat{Cov}(\bar{y}_2, \bar{y}_1)$

Equivalently, if we let $d_i = y_{2i} - y_{1i}$, the variance of $(\bar{y}_2 - \bar{y}_1)$ = variance of d_i. The estimated variance of d_i is:

$$= \frac{\sum_{i=1}^{10} d_i^2 - \frac{\left(\sum_{i=1}^{10} d_i\right)^2}{10}}{(10-1)}$$

Example data with calculated means, variances, and standard errors (SE) are given in Table 1.

Table 1. Example data and calculations of change estimates.

I	y_{1i}	y_{2i}	d_i
1	323.5	391.9	68.4
2	170.6	256.7	86.1
3	412.6	458.4	45.7
4	35.9	91.8	55.9
5	0.0	0.0	0.0
6	237.9	292.9	54.9
7	158.6	230.3	71.7
8	130.1	189.9	59.8
9	0.0	0.0	0.0
10	41.6	114.4	72.8
Mean	151.1	202.6	51.5
Variance	19716.63	23933.97	864.26
SE	1971.66	2393.40	86.43

5. APPENDIX II – EXAMPLES OF GRAPHICAL & STATISTICAL COMPARISONS

5.1 OVERVIEW

The following provides an example of how 30 independent, re-measured randomly located plots in the post-harvest regenerated portion of a coastal management unit could be used to check estimates of volume/ha and volume growth/ha. Possible analyses following three measurements (establishment and two re-measurements) are presented. In this example overall average trends within the management unit are examined. The same procedures could be applied to examine sub-populations within the unit.

5.2 SIMULATED DATA

The following steps were taken to generate the 30 re-measured plots:

1. Each plot was randomly assigned a leading species based on the estimated proportion of the coastal area covered by that leading species.
2. Each plot was randomly assigned an age between 0 – 60 years. A skewed distribution towards the younger ages was assumed.
3. A VDYP yield curve was generated for each leading species assuming an average site index.
4. Each plot was randomly assigned an initial volume (volume at establishment) by selecting the appropriate yield curve and assuming no bias and a CV of 30% around that curve.

Volumes for the first and second re-measurements (5- and 10-years after establishment) were generated by assuming no bias and a CV of 15% around the periodic growth increments and adding these to the previous volume.

This simulated population is post-harvest regenerated stands. The population will expand with time as more area is harvested, and plots would be added to the new portion of the population. However, additional plots established in 5- and 10-years in the new portions of the simulated population were not included in this example. These additional plots would not have any merchantable volumes.

5.3 GRAPHICAL ANALYSES

5.3.1 Volume Versus Age

At each measurement, volume is plotted against stand age; an aggregate yield curve for the management unit is included for comparison Figure 1 shows that the sample data follows the shape of the overall aggregate yield curve.

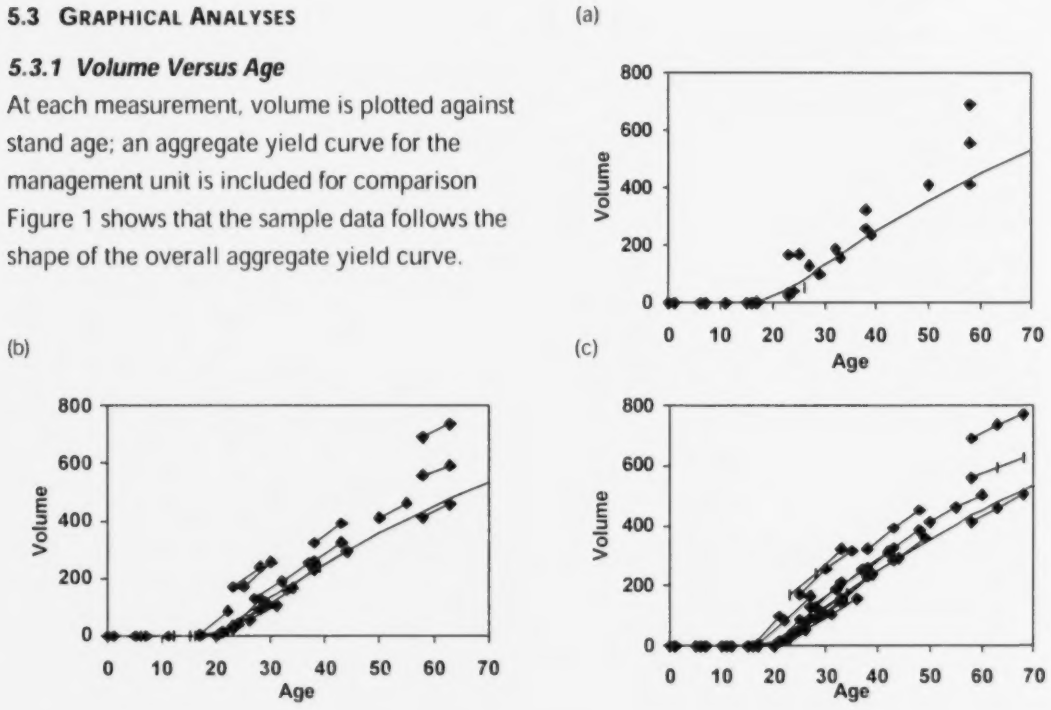

Figure 1. Measured volume versus age at: (a) establishment; (b) first re-measurement; and (c) second re-measurement. The curve plotted is the aggregate yield curve for the management unit.

5.3.2 Actual Versus Predicted

Actual versus predicted volume and volume growth for the sample plots are plotted (Figure 2). It appears that the predicted and observed values are highly correlated.

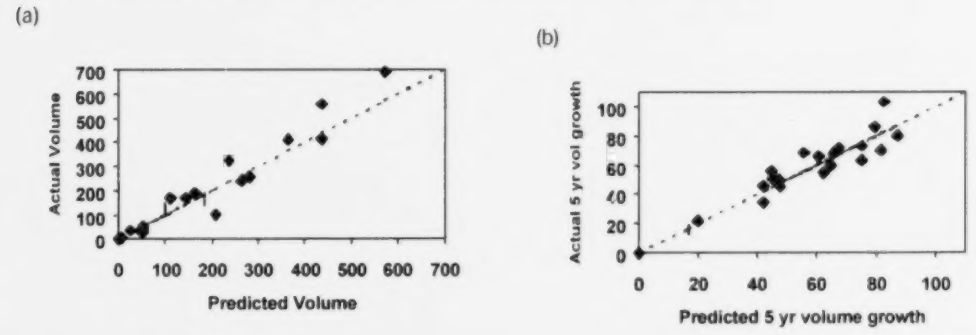

Figure 2. Actual versus predicted yield a) at establishment, and (b) first 5 years growth.

5.3.3 Residuals Versus Age

Absolute or relative differences (residuals) between actual and predicted volume growth are plotted against age (Figure 3). These graphs can be used to decide if observed errors in yield and growth are within reasonable limits and levels-of-comfort at different ages.

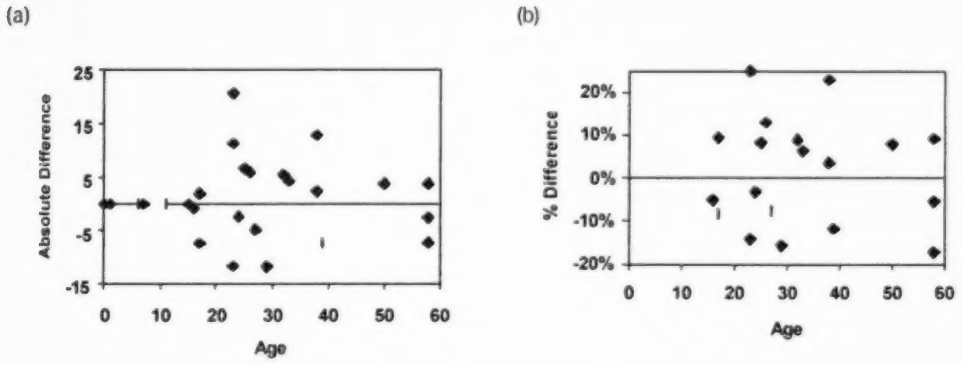

Figure 3. Difference between actual and predicted 5 years growth from establishment to first re-measurement. (a) Absolute difference versus age at establishment, and (b) percent difference (% of predicted) versus age at establishment.

5.4 Statistical Comparisons

Simple statistical tests can follow the graphical analysis. For example, post-stratifying the data by age class and calculating the residual (actual minus predicted growth or yield) statistics (mean and its confidence interval) by age class may provide useful information (Figure 4). The wide confidence intervals in this example are due to the small sample sizes in each age class (stratum).

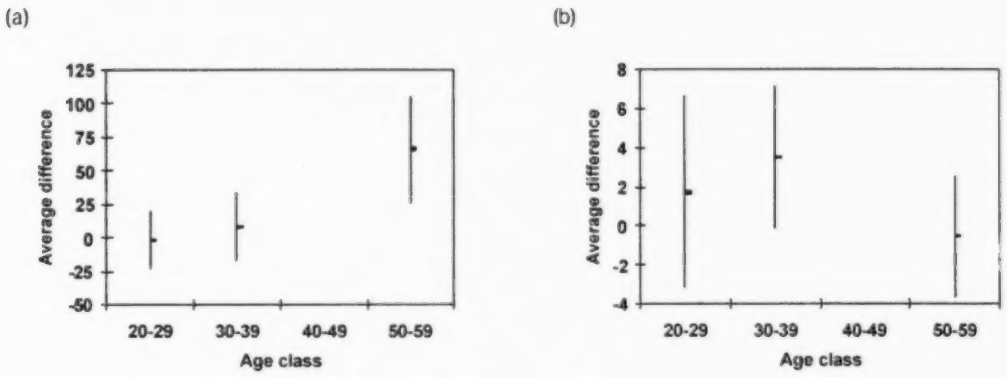

Figure 4. Confidence intervals (95%) on average differences by age class. (a) Average difference between actual and predicted yield at establishment and (b) average difference between actual and predicted growth in the first 5-year period.

6. APPENDIX III – DEFINITIONS

We define the following terms for this report:

Change is *net change,* which is equal to survivor growth plus ingrowth and less mortality.

Change inventory is the process of observing changes and trends over time in the level of the resource and change in land cover classification between two or more time points.

Change projection is the process of predicting the difference in future level or classification of the resource between two or more time points in a management unit.

Change monitoring is an independent check on the projected change or growth in a management unit.

Growth monitoring is the process of observing the growth of a forest and comparing this with the predicted growth of that forest. Growth monitoring is a specific type of *change monitoring.*

Yield audit is the process of observing the yield of a forest and comparing this with the predicted yield of that forest.